

# INTRODUCTION TO FUNCTIONAL DEPENDENCY NETWORK ANALYSIS

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## ABSTRACT

Critical considerations in engineering enterprise systems are identifying, representing, and measuring dependencies between suppliers of technologies and providers of services to consumers and users. The importance of this problem is many-fold. Primary is enabling the study of ripple effects of failure in one capability on other dependent capabilities across the enterprise. Providing mechanisms to anticipate these effects early in design enables engineers to minimize dependency risks that, if realized, can have cascading negative effects on the ability of an enterprise to deliver services to users.

The approach to this problem is built upon concepts from graph theory. Graph theory enables (1) a visual representation of complex interrelationships between entities and (2) the design of analytical formalisms that trace the effects of dependencies between entities as they affect many parts and paths in a graph. In this context, an engineering system is represented as a *directed graph* whose entities are nodes that depict direction, strength, and criticality of supplier-provider relationships. Algorithms are designed to measure capability operability (or inoperability) due to degraded performance (or failure) in supplier and program nodes within capability portfolios that characterize the system.

Capturing and analyzing dependencies is not new in systems engineering. New is tackling this problem (1) in an enterprise systems engineering context where multidirectional dependencies can exist at many levels in a system's capability portfolio and (2) by creating a flexible analysis and measurement approach applicable to any system's capability portfolio, whose supplier-provider relationships can be represented by graph theoretic formalisms.

The methodology is named *Functional Dependency Network Analysis* (FDNA). Its formulation is motivated, in part, by concepts from Leontief systems, the Inoperability Input-Output Model (IIM), Failure Modes and Effects Analysis (FMEA), and Design Structured Matrices (DSM). FDNA is a new analytic approach. One that enables management to study and anticipate the ripple effects of losses in supplier-program contributions on a system's dependent capabilities before risks that threaten these suppliers are realized. An FDNA analysis identifies whether the level of operability loss, if such risks occur, is tolerable. This enables management to better target risk resolution resources to those supplier programs that face high risk and are most critical to a system's operational capabilities.

**KEY WORDS:** Risk, capability risk, capability portfolio, dependencies, operability, inoperability, engineering systems, Leontief matrix, design structured matrix (DSM), failure mode and effects analysis (FMEA), inoperability input-output model (IIM), functional dependency network analysis (FDNA).

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## **1. INTRODUCTION AND BACKGROUND**

Some of the critical and problematic activities in engineering management are identifying, representing, and measuring dependencies between entities involved in engineering an enterprise system. These entities can be generally classified as suppliers of technologies and services and the users or receivers of these technologies and services. The importance of understanding entity relationships is many-fold. Primary is enabling the study of ripple effects of failure of one entity on other dependent entities across the enterprise. Providing mechanisms to anticipate these effects early in a system's design enables engineers to minimize dependency risks that, if realized, may have cascading negative effects on the ability of an enterprise to achieve capability objectives in the delivery of services to users.

In this paper, we approach this topic from the perspective of graph theory. Graph theory enables the visual representation of complex dependencies between entities (i.e. suppliers and users) and enables the design of analytical formalisms that measure and trace these relationships as they affect many parts and paths in a graph.

We assert an enterprise system can be represented as a directed graph whose entities are nodes that depict the direction, strength, and criticality of supplier-provider relationships. Algorithms are then designed to address the question

*What is the effect on the operability of enterprise capability if, due to the realization of risks, one or more contributing programs or supplier-provider chains degrade, fail, or are eliminated?*

The method presented in this paper is called Functional Dependency Network Analysis (FDNA). FDNA enables management to study and anticipate the ripple effects of losses in supplier-program contributions on an enterprise system's dependent capabilities before risks that threaten these suppliers are realized. An FDNA analysis identifies whether the level of operability loss, if such risks occur, is acceptable. This enables management to better target risk resolution resources to those supplier programs that face high risk and are most critical to an enterprise system's operational capabilities.

### *Complex Systems, Systems of Systems, and Enterprise Systems*

In today's literature, the terms complex systems, systems of systems, and enterprise systems are commonly found. What do these terms mean and how are they related? Although these are cutting edge topics in engineering systems and system research, a convergence of thought is beginning to emerge. We begin with the term complex system.

Keating et al (2003) describe complex systems as those having attributes characterized by Jackson (1991). These attributes, paraphrased, include the following:

- Uncountable number of variables or elements that characterize the system
- Unpredictable behaviors among elements with numerous rich interactions
- Purposeful but pluralistic pursuits of multiple goals by providers, users, system entities

Examples of complex systems include the space shuttle, a nuclear power plant, or a magnetic resonance imaging scanner. More recently, and consistent with the above, White (2006) defines a complex system as "an open system with continually cooperating and competing elements – a system that continually

evolves and changes its behavior according to its own condition and its external environment. Changes between states of order and chaotic flux are possible. Relationships between elements are imperfectly known and difficult to understand, predict, or control”.

Engineering systems today are challenged when complex systems become more and more networked in ways that create metasystems – systems of systems “comprised of multiple embedded and interrelated autonomous complex subsystems” [Keating, 2004, 2008]. Similarly, White (2006) defines a system of systems (SoS) as “a collection of systems that function to achieve a purpose not achievable by the individual systems acting independently. Each system can operate independently and accomplish its own separate purpose”. In a system of systems their whole is indeed more than the sum of their parts; however, it can’t exist without them.

Systems of systems form from integrations of multiple subsystems, where each subsystem can be a complex system. Examples of systems of systems include the Ballistic Missile Defense System (BMDS), the international earth observer program known as GEOSS (Global Earth Observation System of Systems), and navigation systems such as the Global Positioning System (GPS). Building systems of systems like these is an enormous engineering and management challenge. If those challenges aren’t enough, engineering systems of networked systems of systems is an even greater challenge and the newest being faced in engineering systems.

Systems of networked systems of systems are sometimes called enterprise systems. Enterprise systems such as the Internet, the Department of Defense Global Information Grid (GIG), or the Federal Aviation Administration’s National Airspace Systems (NAS) are the cutting edge of information age computing and global communications. The literature is very young on engineering enterprise systems. However, scholarship has begun to emerge from academia and industry. Writings by Allen (MIT, 2004) and Rebovich (MITRE, 2005) reflect thought trends from academic and industry perspectives, respectively.

In the monograph “Engineering Systems: An Enterprise Perspective” Allen et al (2004) reflects on the nature of an enterprise and its effects on design and engineering solutions. “Such designs are no longer purely technical. In many cases, the enterprise issues are far more difficult than the technical ones to solve; moreover, there must be adaptation on both sides of the relationship between system and enterprise” [Allen, et al, 2004]. Moreover, Allen identifies the critical and sometimes orthogonal relationships and goals of the multiple stakeholders in the design of an enterprise system. In this monograph Allen writes:

“An enterprise perspective on system design makes us aware of the fact that most such designs engage multiple stakeholders. These can range from shareholders to suppliers to members of the workforce to customers to society. What impact can this far-reaching effect have on system design? First of all, stakeholders’ interests are not always in alignment.

System design may have to take this into account, balancing the interests of the various stakeholders. As a result, the design process is far more complex than one would be led to believe from the engineering science model that we teach to undergraduate engineering students. The best technical solution to a design may very well not be the best overall solution. In fact, it seldom is, and there may not even be a best technical design.

Take the current F-35 aircraft design. With several customers, each having different missions for this system, the designers cannot optimize the design for any one of the customers’ desires. Since recruiting

customers in different countries often means engaging suppliers from those countries, adaptations may need to be made in the design to match the capabilities of those suppliers” [Allen, et al, 2004].

Allen’s insights echoed Bertalanffy’s who recognized that systems, such as the F-35 or Boston’s Big Dig, are fundamentally organizations made of entities (e.g., people) understood by “studying them not in isolation” but in how they assemble, react, and interact as a whole. Rebovich (2005, 2007), and other systems thinkers at MITRE, offer a view on what is meant by an enterprise and what is fundamentally different. They write the following:

“By enterprise we mean a network of interdependent people, processes and supporting technology not fully under control of any single entity. In business literature an enterprise frequently refers to an organization, such as a firm or government agency; in the computer industry it refers to any large organization that uses computers.

Our definition emphasizes the interdependency of individual systems and even systems of systems. We include firms, government agencies, large information-enabled organizations and any network of entities coming together to collectively accomplish explicit or implicit goals. This includes the integration of previously separate units. The enterprise displays new behaviors that emerge from the interaction of the parts” [MITRE, 2007].

#### *What is Different?*

“A mix of interdependency and unpredictability, intensified by rapid technology change, is driving the need for new systems engineering techniques. When large numbers of systems are networked together to achieve some collaborative advantage, interdependencies spring up among the systems. Moreover, when the networked systems are each individually adapting to both technology and mission changes, then the environment for any given system becomes essentially unpredictable. The combination of massive interdependencies and unpredictability is fundamentally different. Systems engineering success is defined not for an individual known system, but for the network of constantly changing systems” [MITRE, 2007].

From this, a key differentiator of an enterprise system is diminished control over its engineering by a centralized authority. Centralized or hierarchical control over design decisions is a feature in engineering systems of systems and traditional, well-bounded, systems (e.g., an airplane or an automobile). Systems of systems are, in most cases, engineered in accordance with stated specifications. These may be shaped by multiple stakeholders, but they are managed by a centralized authority with overall responsibility for engineering and fielding the system of systems.

This is not the case in engineering enterprise systems. An enterprise system is not characterized by firm and fixed specifications under the control of a centralized authority and agreed to by all participants throughout their organizational levels. The envelop that captures stakeholders affected by, or involved with, an enterprise system is so broad that centralized or hierarchical control over its engineering is generally not possible and perhaps not even desirable.

“Enterprise engineering is directed towards enabling and achieving enterprise-level and cross-enterprise operations outcomes. Enterprise engineering is based on the premise that an enterprise is a collection of entities that want to succeed and will adapt as needed. Enterprise engineering processes shape the space in which organizations develop systems, so an organization innovating and operating to succeed in its local mission will simultaneously do so in the interest of the enterprise” [Rebovich, 2007].

It is from this view FDNA was designed. The analytic philosophy was to approach analyses of entity dependencies in an enterprise space from a “whole systems” perspective. A perspective with roots in the writings of Bertalanffy (1968) and one influenced by recognizing the whole of an enterprise is not just more than the sum of its parts – but one wholly and continually shaped, expanded, or diminished by them.

## 2. FUNCTIONAL DEPENDENCY NETWORK ANALYSIS

One way management plans for engineering an enterprise is to create capability portfolios of technology programs and initiatives that, when synchronized, will deliver time-phased capabilities that advance enterprise goals and mission outcomes [Garvey, 2008]. Thus, a capability portfolio is a time dynamic organizing construct to deliver capabilities across specified epochs [Garvey, 2008].

Creating capability portfolios is a complex management and engineering analysis activity. Once a capability portfolio’s hierarchy and its elements are “defined” it is managed by a team to ensure its collection of technology programs and technology initiatives combine in ways to deliver one or more capabilities to the enterprise. Thus, one can take a supplier-provider view of a capability portfolio.

In this context, a capability portfolio is represented as a directed graph whose entities are nodes that depict the direction, strength, and criticality of supplier-provider dependency relationships. Algorithms are designed to measure levels of capability inoperability due to degraded performance (or failure) in supplier and program nodes where mutual relationships exist between them.

We approach this problem from the perspective of graph theory. Graph theory enables (1) a visual representation of complex interrelationships between entities and (2) the design of a new calculus that provides a way to measure and trace the effects of dependencies between entities as they affect many parts and paths in a graph. The idea behind FDNA is best illustrated by the graph in Exhibit 1. Here, a simple capability portfolio is shown. It consists of three capability nodes and six program nodes. Mathematically, Exhibit 1 illustrates a special type of graph known as a directed graph.

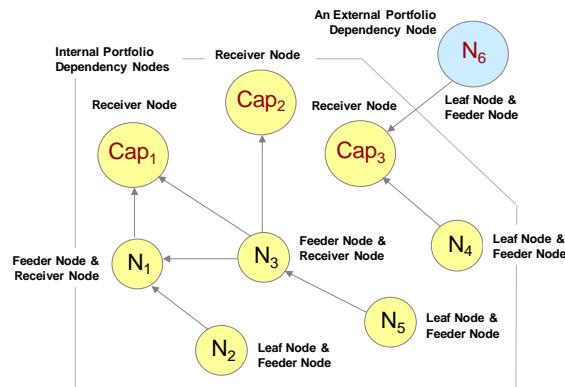


Exhibit 1  
 An FDNA Graph: A Capability Portfolio Context

A graph may also be viewed in terms of parent-child relationships. A parent node is one with lower-level nodes coming from it. These nodes are called child nodes to that parent node. Nodes that terminate in a network are sometimes called leaf-nodes. Leaf-nodes are terminal nodes in that they have no children coming from them [Garvey, 2008].

*FDNA Fundamentals*

We begin with a discussion of dependence and what it means in the FDNA methodology. In an FDNA graph, dependence is a condition that exists between two nodes when the operability of one node relies, to some degree, on the operability of another node. This is illustrated in Exhibit 2.

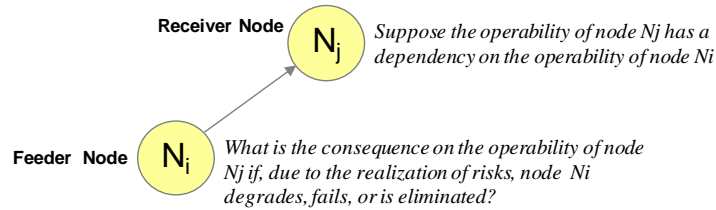


Exhibit 2  
 An FDNA Graph Models Dependency Relationships

What is meant by operability? Operability is a state where a node is functioning at some level of performance. The level of performance achieved by a node can be expressed by a measure of value, worth, or “the utility it yields” [Bernoulli, 1738]. In FDNA, a node’s measure of value is called its operability level. Value is analogous to a von Neumann-Morgenstern (vNM) utility, a dimensionless number often expressed in “utils”. In FDNA, we define what a node produces as its measure of performance (MOP) and the value of what is produced as its operability level or its measure of effectiveness (MOE).

In a dependency relationship between nodes, contributions to the dependent node from other nodes are context specific to the natures of the supplying nodes. Contributions result from the achievement of outputs by nodes that reflect their performance. For example, suppose node  $N_i$  produces and supplies coolant fluid to various engine manufacturers. A measure of performance for this node might be the rate with which it produces coolant fluid. Suppose a production rate of 9000 gallons per hour means this node is performing at half its full operability level. If one-hundred utils means a node is wholly operable and zero utils means a node is wholly inoperable, then a performance level of 9000 gallons per hour is worth 50 utils of value. We can continue to assess the value (or worth) of other performance levels achieved by  $N_i$  to form what is known as a value function. Exhibit 3 illustrates such a function.

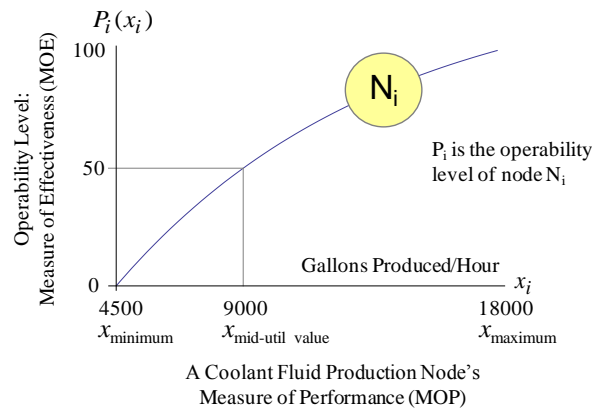


Exhibit 3  
 An Example Value Function for the Operability Level of Node  $N_i$

An FDNA graph can be viewed as a topology of receiver-feeder node relationships. A receiver node is one whose operability level relies, to some degree, on the operability level of at least one feeder node. In FDNA, a node may be a feeder and a receiver node as shown in Exhibit 4.

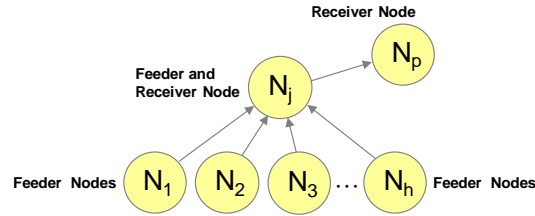


Exhibit 4  
 An FDNA Graph is a Topology of Receiver-Feeder Nodes

In FDNA, a receiver node's operability level is influenced by two properties of dependency. The first is the strength with which a receiver node's operability level relies on the operability level of feeder nodes. The second is the criticality of feeder node contributions to a receiver node for it to achieve its operability level objectives. Respectively, we call these the Strength of Dependency (SOD) and the Criticality of Dependency (COD) constraint. These concepts are illustrated in the following narrative.

*In Exhibit 2, suppose receiver node  $N_j$  is a widget production machine and feeder node  $N_i$  supplies coolant and lubricant fluids for the machine's engine. Suppose the machine is wholly operable when it is producing 120 widgets per hour; that is,  $P_j(x_j = 120) = 100$  utils. Without any supply of fluids from  $N_i$  suppose  $N_j$  can only produce 80 widgets per hour. Suppose a production rate of 80 widgets per hour is worth 55 utils; that is,  $P_j(x_j = 80) = 55$  utils when  $P_i = 0$ . This implies the baseline operability level of  $N_j$ , denoted by  $BOLP_j$ , is 55 utils. Suppose the fluids from supplier node  $N_i$  are ideal for lowering the operating temperature of the engine and increasing the output of the widget production machine. Without these fluids the engine's temperature will rise, its parts will wear, and the machine will decline from its baseline operability level of 55 utils and eventually become wholly inoperable (0 utils).*

From this, we see the widget production machine not only relies on the performance of its fluids supplier to improve its baseline production output, but it has a critical dependency on this relationship to ensure its operational effectiveness. Thus, SOD and COD capture different but important effects of feeder-receiver node relationships on their operability levels. Where SOD captures the effects of relationships that improve baseline operability levels, COD captures whether such relationships could involve losses or constraints on these levels. FDNA permits this loss-gain dualism to compete within its calculus – a dependency dualism that models a mix of positive or negative effects that complex receiver-feeder node interactions can have across a topology of multi-nodal relationships.

From this, the operability level of a receiver node  $N_j$  that depends on the operability level of feeder node  $N_i$  can be expressed by the general function

$$P_j = f(\alpha_{ij}, \beta_{ij}, P_i), 0 \leq \alpha_{ij} \leq 1, 0 \leq \beta_{ij} \leq 100, 0 \leq P_i, P_j \leq 100 \quad (1)$$

where  $P_j$  is the operability level of  $N_j$ ,  $\alpha_{ij}$  is the strength of dependency fraction,  $\beta_{ij}$  is the criticality of dependency constraint, and  $P_i$  is the operability level of  $N_i$ .

If, in Equation 1, the function  $f$  is defined by a principle known as the weakest link then

$$P_j = \text{Min}(g(\alpha_{ij}, P_i), h(\beta_{ij}, P_i)) \quad 0 \leq P_i, P_j \leq 100 \quad (2)$$

One form of the function  $g$  that satisfies the above discussion about  $\alpha_{ij}$  is

$$g(\alpha_{ij}, P_i) = \text{SODP}_j = \alpha_{ij}P_i + 100(1 - \alpha_{ij}) \quad (3)$$

One form of the function  $h$  that satisfies the above discussion about  $\beta_{ij}$  is

$$h(\beta_{ij}, P_i) = \text{CODP}_j = P_i + \beta_{ij} \quad (4)$$

More generally, the operability level of node  $N_j$  that is dependent on the operability levels of  $h$  feeder nodes  $N_1, N_2, N_3, \dots, N_h$  is

$$0 \leq P_j = \text{Min}(\text{SODP}_j, \text{CODP}_j) \leq 100 \quad (5)$$

where

$$\text{SODP}_j = \text{Average}(\text{SODP}_{j1}, \text{SODP}_{j2}, \text{SODP}_{j3}, \dots, \text{SODP}_{jh})$$

$$\text{SODP}_{ji} = \alpha_{ij}P_i + 100(1 - \alpha_{ij})$$

$$\text{CODP}_j = \text{Min}(\text{CODP}_{j1}, \text{CODP}_{j2}, \text{CODP}_{j3}, \dots, \text{CODP}_{jh})$$

$$\text{CODP}_{ji} = P_i + \beta_{ij}$$

and  $0 \leq \alpha_{ij} \leq 1, 0 \leq \beta_{ij} \leq 100, 0 \leq P_i, P_j \leq 100, i = 1, 2, 3, \dots, h$

#### Assessing $\alpha_{ij}$ and $\beta_{ij}$

There are many ways to determine strength and criticality of dependency between nodes in an FDNA graph. This discussion illustrates a few approaches. We begin with determining the strength of dependency parameter  $\alpha_{ij}$ . This is followed by a discussion on the criticality constraint  $\beta_{ij}$ .

A receiver node's baseline operability level can be used to determine  $\alpha_{ij}$ . With this, we can ask the following: *What is the receiver node's baseline operability level (utils) prior to receiving its feeder node's contribution?* If the answer is 0 utils, then  $\alpha_{ij} = 1$ . If the answer is 50 utils, then  $\alpha_{ij} = 0.50$ . If the answer is 70 utils, then  $\alpha_{ij} = 0.30$  and so forth. Thus,  $\alpha_{ij}$  can be solved from the expression

$$100(1 - \alpha_{ij}) = x$$

where  $x$  is the receiver node's baseline operability level prior to receiving its feeder node's contribution. The greater the value of  $\alpha_{ij}$  the greater the strength of dependency that receiver node  $N_j$  has on feeder node  $N_i$  and the less  $N_j$ 's operability level is independent of  $N_i$ 's level. The smaller the value of  $\alpha_{ij}$  the lesser the strength of dependency that receiver node  $N_j$  has on feeder node  $N_i$  and the more  $N_j$ 's operability level is independent of  $N_i$ 's level. Next, we present a way to assess criticality of dependency.

Criticality of dependency (COD) enables the operability level of a receiver node to be constrained by the operability levels of its feeder nodes. This allows a receiver node's operability level to be limited by the performance of one feeder node, if appropriate, even when the receiver's other feeder nodes are wholly operable. In general, the criticality of dependency constraint is the operability level  $\beta_{ij}$  (utils) such that the operability level of receiver node  $N_j$  with  $h$  feeder nodes can never be more than  $P_i + \beta_{ij}$  for all  $i$ , where  $i = 1, 2, 3, \dots, h$ ,  $0 \leq \beta_{ij} \leq 100$ , and  $P_i$  is the operability level of feeder node  $N_i$ .

We can also characterize this constraint in terms of degradation in a receiver node's operability level, where degradation is measured from an operability level that has meaning with respect to the receiver node's performance goals or requirements. For example, in a single feeder-receiver node pair (as in Exhibit 2) the criticality of dependency constraint can be viewed as the operability level  $\beta_{ij}$  that receiver node  $N_j$  degrades to from a reference point operability level (such as its baseline operability level) when its feeder node's performance level has zero operational utility (no value or worth) to  $N_j$ .

#### Formulating FDNA Equations

The following illustrates how FDNA equations are formulated from an FDNA graph. The approach shown scales to an FDNA graph of any complexity. Consider Exhibit 5.

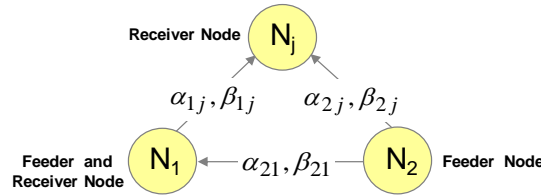


Exhibit 5  
 A 3-Node FDNA Graph with 3-Dependency Points and 2-Receiver Nodes

The FDNA equations for the relationships in Exhibit 5 are as follows:

$$P_j = \text{Min} \left( \frac{\alpha_{1j}P_1}{2} + \frac{\alpha_{2j}P_2}{2} + 100 \left( 1 - \left( \frac{\alpha_{1j} + \alpha_{2j}}{2} \right) \right), P_1 + \beta_{1j}, P_2 + \beta_{2j} \right)$$

$$P_1 = \text{Min}(\alpha_{21}P_2 + 100(1 - \alpha_{21}), P_2 + \beta_{21})$$

where

$\alpha_{1j}$  is the strength of dependency fraction between  $N_1$  and  $N_j$ ,  $0 \leq \alpha_{1j} \leq 1$

$\alpha_{2j}$  is the strength of dependency fraction between  $N_2$  and  $N_j$ ,  $0 \leq \alpha_{2j} \leq 1$

$\alpha_{21}$  is the strength of dependency fraction between  $N_1$  and  $N_2$ ,  $0 \leq \alpha_{21} \leq 1$

$\beta_{1j}$  is the criticality of dependency constraint between  $N_1$  and  $N_j$ ,  $0 \leq \beta_{1j} \leq 100$

$\beta_{2j}$  is the criticality of dependency constraint between  $N_2$  and  $N_j$ ,  $0 \leq \beta_{2j} \leq 100$

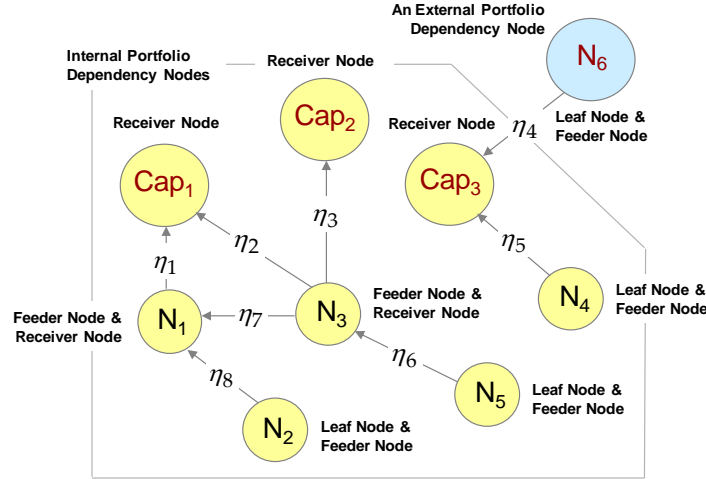
$\beta_{21}$  is the criticality of dependency constraint between  $N_1$  and  $N_2$ ,  $0 \leq \beta_{21} \leq 100$

$0 \leq P_1, P_2, P_j \leq 100$

In an FDNA graph, the number of dependency equations equals the number of receiver nodes. The graph in Exhibit 5 has two receiver nodes. These are  $N_j$  and  $N_1$ . The operability level of  $N_j$  is a function of the operability level of  $N_1$  and the operability level of  $N_2$ . The operability level of  $N_1$  is also a function of the operability level of  $N_2$ . Hence, we have two dependency equations for the FDNA graph in Exhibit 5.

### Computational Illustration

This section presents an FDNA operability analysis of the graph in Exhibit 1, which is shown below as Exhibit 6 for convenience.



$$\begin{aligned} \eta_1 &\triangleq \alpha_{1Cap_1}, \beta_{1Cap_1} & \eta_2 &\triangleq \alpha_{3Cap_1}, \beta_{3Cap_1} & \eta_3 &\triangleq \alpha_{3Cap_2}, \beta_{3Cap_2} & \eta_4 &\triangleq \alpha_{6Cap_3}, \beta_{6Cap_3} \\ \eta_5 &\triangleq \alpha_{4Cap_3}, \beta_{4Cap_3} & \eta_6 &\triangleq \alpha_{53}, \beta_{53} & \eta_7 &\triangleq \alpha_{31}, \beta_{31} & \eta_8 &\triangleq \alpha_{21}, \beta_{21} \end{aligned}$$

Exhibit 6  
 An FDNA Graph: A Capability Portfolio Context

The FDNA equations for the relationships in Exhibit 6 are as follows:

$$P_1 = \text{Min} \left( \frac{\alpha_{21}P_2}{2} + \frac{\alpha_{31}P_3}{2} + 100 \left( 1 - \left( \frac{\alpha_{21} + \alpha_{31}}{2} \right) \right), P_2 + \beta_{21}, P_3 + \beta_{31} \right)$$

$$P_3 = \text{Min}(\alpha_{53}P_5 + 100(1 - \alpha_{53}), P_5 + \beta_{53})$$

$$P_{Cap_1} = \text{Min} \left( \frac{\alpha_{1Cap_1}P_1}{2} + \frac{\alpha_{3Cap_1}P_3}{2} + 100 \left( 1 - \left( \frac{\alpha_{1Cap_1} + \alpha_{3Cap_1}}{2} \right) \right), P_1 + \beta_{1Cap_1}, P_3 + \beta_{3Cap_1} \right)$$

$$P_{Cap_2} = \text{Min}(\alpha_{3Cap_2}P_3 + 100(1 - \alpha_{3Cap_2}), P_3 + \beta_{3Cap_2})$$

$$P_{Cap_3} = \text{Min} \left( \frac{\alpha_{4Cap_3}P_4}{2} + \frac{\alpha_{6Cap_3}P_6}{2} + 100 \left( 1 - \left( \frac{\alpha_{4Cap_3} + \alpha_{6Cap_3}}{2} \right) \right), P_4 + \beta_{4Cap_3}, P_6 + \beta_{6Cap_3} \right)$$

From these equations, Exhibit 7 presents the resulting operability analysis as feeder nodes in Exhibit 6 lose operability (say) over time periods t1, t2, and t3.

FUNCTIONAL DEPENDENCY NETWORK ANALYSIS (FDNA)							
A CAPABILITY PORTFOLIO							
<b>INPUT: <math>\alpha_{ij}</math> Strength of Dependency (SOD)</b>							
$\alpha_{1Cap1}$	0.90	$\alpha_{4Cap3}$	0.85				
$\alpha_{3Cap1}$	0.45	$\alpha_{53}$	0.30				
$\alpha_{3Cap2}$	0.65	$\alpha_{31}$	0.15				
$\alpha_{6Cap3}$	0.90	$\alpha_{21}$	0.28				
<b>INPUT: <math>\beta_{ij}</math> Criticality of Dependency (COD)</b>							
$\beta_{1Cap1}$	10.00	$\beta_{4Cap3}$	15.00				
$\beta_{3Cap1}$	55.00	$\beta_{53}$	70.00				
$\beta_{3Cap2}$	35.00	$\beta_{31}$	85.00				
$\beta_{6Cap3}$	10.00	$\beta_{21}$	72.00				
<b>INPUT: IF these feeder nodes are functioning at these operability levels ...</b>							
Time t1: If operability levels of feeder nodes P2, P5, P4, and P6 are:		Time t2: If operability levels of feeder nodes P2, P5, P4, and P6 are:			Time t3: If operability levels of feeder nodes P2, P5, P4, and P6 are:		
P2	100	P2	75	P2	50		
P5	100	P5	75	P5	50		
P4	100	P4	75	P4	50		
P6	100	P6	100	P6	100		
<b>OUTPUT: Then these receiver nodes are functioning at these operability levels...</b>							
P3	100.00	P3	92.50	P3	85.00		
P1	100.00	P1	95.94	P1	91.88		
PCap1	100.00	PCap1	96.48	PCap1	92.97		
PCap2	100.00	PCap2	95.13	PCap2	90.25		
PCap3	100.00	PCap3	89.38	PCap3	65.00		
<b>COD portion of receiver node operability level</b>							
P3	170.00	P3	145.00	P3	120.00		
P1	172.00	P1	147.00	P1	122.00		
PCap1	110.00	PCap1	105.94	PCap1	101.88		
PCap2	135.00	PCap2	127.50	PCap2	120.00		
PCap3	110.00	PCap3	90.00	PCap3	65.00		
<b>SOD portion of receiver node operability level</b>							
P3	100.00	P3	92.50	P3	85.00		
P1	100.00	P1	95.94	P1	91.88		
PCap1	100.00	PCap1	96.48	PCap1	92.97		
PCap2	100.00	PCap2	95.13	PCap2	90.25		
PCap3	100.00	PCap3	89.38	PCap3	78.75		

Exhibit 7  
 An FDNA Operability Analysis of the Capability Portfolio in Exhibit 6

### 3. CONCLUSION

This paper presented a brief introduction to Functional Dependency Network Analysis (FDNA). FDNA was developed to model and measure dependency relationships between suppliers of technologies and providers of services these technologies enable the enterprise to deliver.

The importance of the dependency problem in enterprise engineering is many-fold. Primary is enabling the study of ripple effects of failure in one capability on the operability of other dependent capabilities across an enterprise. Providing mechanisms to anticipate these effects early in design enables engineers to minimize dependency risks that, if realized, may have cascading negative effects on the ability of an enterprise to deliver services to users.

The motivation for FDNA came from the need to address dependency problems that could not be fully expressed or solved in matrix-based protocols, such as those that characterize input-output (I/O) models in economic science. FDNA equations are constructed from mathematical graphs in ways that enable solutions to be derived by a composition of functions; that is, FDNA equations are algebraically formulated by a composition of functional dependency relationships across a mathematical graph. This strategy avoids matrix algebra and linear system solution issues (such as stability) that sometimes arise in matrix-based input-output approaches.

The FDNA structure is visualized by graph theory to represent and model a range of complex dependency relationships between entities. FDNA has the potential to be a generalized modeling approach for a variety of dependency problems, including those in the domains of input-output economics, critical infrastructure risk analysis, and non-stationary, temporal, dependency analysis problems.

Additional research areas include the following:

- **Analytical Scalability:** Research how to approach risk analysis in engineering enterprise systems that consist of dozens of capability portfolios with hundreds of supplier programs. Explore representing large-scale enterprises by domain capability portfolio clusters and investigate a concept for portfolio cluster risk management.
- **Non-stationary Considerations:** Extend the FDNA calculus to address non-stationary, temporal, dependency analysis problems. Explore how FDNA can expand and integrate into time-varying modeling and simulation environments, such as those in systems dynamics methods and tools.
- **Optimal Adaptive Strategies:** Research how to optimally adapt an engineering system's supplier-provider network to reconfigure its nodes to maintain operability if risks that threaten these nodes are realized. Consider this problem in stationary and non-stationary perspectives.

In summary, FDNA is a methodology that enables management to study and anticipate the ripple effects of losses in supplier-program contributions on dependent capabilities before risks that threaten these suppliers are realized. FDNA identifies whether the level of operability loss, if such risks occur, is acceptable. This enables management to better target risk resolution resources to those supplier programs that face high risk and are most critical to the operational capabilities of a portfolio.

## REFERENCES AND RELEVANT LITERATURE

- Allen, T., Nightingale, D., Murman, E., March 2004. "Engineering Systems an Enterprise Perspective", An Engineering Systems Monograph, Engineering Systems Division, The Massachusetts Institute of Technology.
- Arrow, K. J., 1965. "Aspects of the Theory of Risk Bearing", Yrjo Jahnsson Lectures, Helsinki, Finland: Yrjo Jahnssonin Saatio.
- Ayyub, B. M., 2001. Elicitation of Expert Opinions for Uncertainty and Risks, Chapman-Hall/CRC-Press, Taylor & Francis Group (UK), Boca Raton, London, New York.
- Ayyub, B. M., McGill, W. L., Kaminsky, M., 2007. "Critical Asset and Portfolio Risk Analysis: An All-Hazards Framework", Risk Analysis, Vol. 27, No. 4.
- Bahnmaier, W. W., editor, 2003. Risk Management Guide for DOD Acquisition, 5th Edition, Version 2.0, Department of Defense Acquisition University Press, Fort Belvoir, Virginia, 22060-5565.
- Bernoulli, D., 1738. "Exposition of a New Theory on the Measurement of Risk", *Econometrica*, Vol. 22, No. 1 (Jan., 1954), pp. 23-36 Virginia, 22060-5565, The Econometric Society, [www.jstor.org/stable/1909829](http://www.jstor.org/stable/1909829).
- Blanchard, B. S., Fabrycky W. J., 1990. Systems Engineering and Analysis, 2nd ed. Englewood Cliffs, New Jersey, Prentice-Hall, Inc.
- Browning, T. R., Deyst, J. J., Eppinger, S. D., 2002. "Adding Value in Product Development by Creating Information and Reducing Risk", *IEEE Transactions on Engineering Management*, Vol. 49, No. 4.
- Chytka, T., Conway, B., Keating, C., Unal, R., 2004. "Development of an Expert Judgment Elicitation And Calibration Methodology for Risk Analysis in Conceptual Vehicle Design", Old Dominion University Project Number: 130012, NASA Grant NCC-1-02044, NASA Langley Research Center, Hampton, Virginia 23681.
- Clemen, R. T., 1996. Making Hard Decisions An Introduction to Decision Analysis, 2nd edition, Pacific Grove, California, Brooks/Cole Publishing Company.
- Cox, L. A., Babayev, D., Huber, W., 2005. "Some Limitations of Qualitative Risk Rating Systems" *Risk Analysis*, Vol. 25, No. 3.
- Cox, L. A., 2009. "Improving Risk-Based Decision Making for Terrorism Applications", *Risk Analysis*, Vol. 29, No. 3.
- Creswell, J. W., 2003. Research Design: Qualitative, Quantitative, and Mixed Methods Approaches (2nd ed.), Sage University Press, Thousand Oaks, California.
- Crowther, K. G., Haimes, Y. Y., Taub, G., 2007. "Systemic Valuation of Strategic Preparedness Through Application of the Inoperability Input-Output Model with Lessons Learned from Hurricane Katrina", *Risk Analysis*, Vol. 27, No. 5.
- Daniels, C. B. and LaMarsh, W. J., 2007. "Complexity as a Cause of Failure in Information Technology Project Management", *Proceedings of IEEE International Conference on System of Systems Engineering*, April, pp.1-7.
- de Finetti, B., 1974. Theory of Probability, Vol. 1., John Wiley & Sons, New York, NY.
- de Finetti, B (author)., A. Mura, A. (editor), 2008. Philosophical Lectures on Probability: Springer-Science + Business Media B. V.

- Dyer, J. S., Sarin, R. K., 1979. "Measurable Multiattribute Value Functions", *Operations Research*, Vol. 27, No. 4, July-August.
- Edwards, J. E., Scott, J. C., Nambury, R. S., 2003. *The Human Resources Program-Evaluation Handbook*, Sage University Press, Thousand Oaks, California.
- Edwards, W., 1954. "The Theory of Decision Making", *Psychological Bulletin*, 41, 380-417.
- Edwards, W., 1961. "Behavioral Decision Theory", *Annual Review of Psychology*, 12, 473-498.
- Fishburn, P. C., "Foundations of Decision Analysis: Along the Way", *Management Science*, Vol. 35, No. 4, April 1989.
- GAO: Government Accountability Office, July 2004. "Defense Acquisitions: The Global Information Grid and Challenges Facing its Implementation", GAO-04-858.
- Garvey, P. R., Cho, C. C., Giallombardo, R., 1997. "RiskNav: A Decision Aid for Prioritizing, Displaying, and Tracking Program Risk", *Military Operations Research*, V3, N2.
- Garvey, P. R., 1999. "Risk Management", *Encyclopedia of Electrical and Electronics Engineering*, John Wiley & Sons, New York, NY.
- Garvey, P. R., 2000. *Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective*, Chapman-Hall/CRC-Press, Taylor & Francis Group (UK), London, Boca Raton, New York; ISBN 0824789660.
- Garvey, P. R., 2001. "Implementing a Risk Management Process for a Large Scale Information System Upgrade – A Case Study", *INSIGHT*, Vol. 4, Issue 1, International Council on Systems Engineering (INCOSE).
- Garvey, P. R., Cho, C. C., 2003. "An Index to Measure a System's Performance Risk", *The Acquisition Review Quarterly (ARQ)*, Vol. 10, No. 2.
- Garvey, P. R., Cho, C. C., 2005. "An Index to Measure and Monitor a System of Systems' Performance Risk", *The Acquisition Review Journal (ARJ)*.
- Garvey, P. R., 2005. "System of systems Risk Management Perspectives on Emerging Process and Practice", *The MITRE Corporation*, MP 04B0000054.
- Garvey, P. R., 2008. *Analytical Methods for Risk Management: A Systems Engineering Perspective*, Chapman-Hall/CRC-Press, Taylor & Francis Group (UK), London, Boca Raton, New York; ISBN 1584886374.
- Garvey, P. R., 2009. *An Analytical Framework and Model Formulation for Measuring Risk in Engineering Enterprise Systems: A Capability Portfolio Perspective*, Ph.D. Dissertation, Old Dominion University, United States, Virginia. August, 2009, Dissertations & Theses, Old Dominion University Library, Publication No. AAT 3371504, ISBN: 9781109331325, ProQuest ID: 1863968631.
- Gelinas, N., 2007. "Lessons of Boston's Big Dig", *City Journal*.
- Gharajedaghi, J., 1999. *Systems Thinking Managing Chaos and Complexity – A Platform for Designing Business Architecture*, Woburn, Massachusetts, Butterworth-Heinemann.
- Haimes, Y. Y., 2004. *Risk Modeling, Assessment, and Management*, 2nd ed., John Wiley & Sons, New York, NY.
- Hansson, S. O., "Risk", *The Stanford Encyclopedia of Philosophy* (Winter 2008 Edition), Edward N. Zalta (ed.), URL = <<http://plato.stanford.edu/archives/win2008/entries/risk/>>.

- Hofstetter, P., Bare, J. C., Hammitt, J. K., Murphy, P. A., Rice, G. E., 2002. "Tools for Comparative Analysis of Alternatives: Competing or Complementary Perspectives?" *Risk Analysis*, Vol. 22, No. 5.
- Hwang, Ching-Lai, Yoon, K. Paul, 1995. *Multiple Attribute Decision Making: An Introduction*, Sage University Paper Series in Quantitative Applications in the Social Sciences, 07-104, Thousand Oaks, California, copyright 1995, by Sage.
- Jackson, M. C., 1991. *Systems Methodology for the Management Sciences*, New York: Plenum.
- Jaynes, E. T., 1988. "Probability Theory as Logic", Ninth Annual Workshop on Maximum Entropy and Bayesian Methods, Dartmouth College, New Hampshire, August 14, 1989. In the Proceedings Volume, *Maximum Entropy and Bayesian Methods*, Paul F. Fougere, Editor, Kluwer Academic Publishers, Dordrecht, Holland (1990).
- Jiang, P., Haimes, Y. Y., 2004. "Risk Management for Leontief-Based Interdependent Systems", *Risk Analysis*, Vol. 24, No. 5.
- Kaplan, S., Garrick, B., 1981. "On the Quantitative Definition of Risk", *Risk Analysis*, Vol. 1, No. 1, pp.11-27.
- Kaplan, S., 1997. "The Words of Risk Analysis", *Risk Analysis*, Vol. 4, No. 17.
- Keating, C., Rogers, R., Unal, R., Dryer, D., Sousa-Poza, A., Safford, R., Peterson, W., Rabadi, G., 2003. "System of Systems Engineering", *Engineering Management Journal*, Vol. 15, No. 3.
- Keating, C. B., Sousa-Poza, A., Mun, Ji Hyon, 2004. "System of Systems Engineering Methodology", Department of Engineering Management and Systems Engineering, Old Dominion University, ©2004, All rights reserved.
- Keating, C., Sousa-Poza, A., Kovacic, S., 2008. "System of Systems Engineering: An Emerging Multidiscipline", *Int. J. System of Systems Engineering*, Vol. 1, Nos. 1/2, pp. 1-17.
- Keeney, R. L., Raiffa, H., 1976. *Decisions with Multiple Objectives Preferences and Value Tradeoffs*, John Wiley & Sons, New York, NY.
- Keeney, R. L., 1992. *Value-Focused Thinking A Path to Creative Decision Making*, Harvard University Press, Cambridge, Massachusetts.
- Kirkwood, C. W., 1997. *Strategic Decision Making: Multiobjective Decision Analysis With Spreadsheets*, California, Duxbury Press.
- Krantz, D. H., Luce, R. D., Suppes, P., Tversky, A., 1971. *Foundations of Measurement, Additive and Polynomial Representations*, Volume 1., New York, Academic Press, Dover Publications.
- Leontief, W. W., 1966. *Input-Output Economics*, Oxford University Press, New York, NY.
- Lian, C., Santos, J. R., Haimes, Y. Y., 2007. "Extreme Risk Analysis of Interdependent Economic and Infrastructure Sectors", *Risk Analysis*, Vol. 27, No. 4.
- Malczewski, J., 1999. *GIS and Multicriteria Decision Analysis*, John Wiley & Sons, New York, NY.
- Mariampolski, H., 2001. *Qualitative Market Research: A Comprehensive Guide*, Sage University Press, Thousand Oaks, California.
- Massachusetts Turnpike Authority (MTA), Big Dig, retrieved from <http://www.massturnpike.com/bigdig/background/facts.html>.

MITRE: 2007. "Evolving Systems Engineering", © 2007, The MITRE Corporation, All Rights Reserved, Distribution Unlimited, Case Number 07-1112.

Moynihan, R. A., 2005. "Investment Analysis Using the Portfolio Analysis Machine (PALMA) Tool", The MITRE Corporation, [www.mitre.org/work/tech\\_papers/tech\\_papers\\_05/05\\_0848/05\\_0848.pdf](http://www.mitre.org/work/tech_papers/tech_papers_05/05_0848/05_0848.pdf).

Moynihan, R. A., Reining, R. C., Salamone, P. P., Schmidt, B. K., 2008. "Enterprise Scale Portfolio Analysis at the National Oceanic and Atmospheric Administration (NOAA)", Systems Engineering, International Council on Systems Engineering (INCOSE), 11 September 2008, © 2008 Wiley Periodicals, Inc.; [www3.interscience.wiley.com/journal/121403613/references](http://www3.interscience.wiley.com/journal/121403613/references).

Murphy, C., Gardoni, P., 2006. "The Role of Society in Engineering Risk Analysis: A Capabilities-Based Approach", Risk Analysis, Vol. 26, No. 4.

Nau, R. F., 2002. "de Finetti Was Right: Probability Does Not Exist", Theory and Decision 51: 89-124, 2001, ©2002, Kluwer Academic Publishers.

National Transportation Safety Board, 2007. Public Meeting, 10 July 2007; "Highway Accident Report: Ceiling Collapse in the Interstate 90 Connector Tunnel", Boston, Massachusetts, NTSB/HAR-07/02.

Office of the Secretary of Defense (OSD), 2005: Net-Centric Operational Environment Joint Integrating Concept, Version 1.0, Joint Chiefs of Staff, 31 October 2005, Joint Staff, Washington, D.C. 20318-6000; [www.dod.mil/cio-nii/docs/netcentric\\_jic.pdf](http://www.dod.mil/cio-nii/docs/netcentric_jic.pdf).

Pinto, C. A., Arora, A., Hall, D., Ramsey, D., Telang, R., 2004. "Measuring the Risk-Based Value of IT Security Solutions", IEEE IT Professional, v.6 no.6, pp. 35-42.

Pinto, C. A., Arora, A., Hall, D., Schmitz, E., 2006. "Challenges to Sustainable Risk Management: Case Example in Information Network Security", Engineering Management Journal, v.18, no.1, pp. 17-23.

Pratt, J. W., 1965. "Risk Aversion in the Small and in the Large", Econometrica, Vol. 32.

Ramsey, F. P. (author), Mellor, D. H. (editor), 1990. "F. P. Ramsey: Philosophical Papers", Cambridge University Press.

Rebovich, G., Jr., 2007. "Engineering the Enterprise", The MITRE Corporation; [www.mitre.org/work/tech\\_papers/tech\\_papers\\_07/07\\_0434/07\\_0434.pdf](http://www.mitre.org/work/tech_papers/tech_papers_07/07_0434/07_0434.pdf).

Rebovich, G., Jr., 2005. "Enterprise Systems Engineering Theory and Practice, Volume 2, Systems Thinking for the Enterprise New and Emerging Perspectives", The MITRE Corporation; [www.mitre.org/work/tech\\_papers/tech\\_papers\\_06/05\\_1483/05\\_1483.pdf](http://www.mitre.org/work/tech_papers/tech_papers_06/05_1483/05_1483.pdf).

Reilly, J., Brown, J., 2004. "Management and Control of Cost and Risk for Tunneling and Infrastructure Projects", Proc. International Tunneling Conference, Singapore.

Rescher, N., 2006. Philosophical Dialectics: An Essay on Metaphilosophy, SUNY Press, Albany, New York.

Rittel, H., 1972. "On the Planning Crisis: Systems Analysis of the First and Second Generations" The Institute of Urban and Regional Development, Reprint No. 107, University of California, Berkeley.

Santos, J. R., Haimes, Y. Y., 2004. "Modeling the Demand Reduction Input-Output (I-O) Inoperability Due to Terrorism of Interconnected Infrastructures", Risk Analysis, Vol. 24, No. 6.

Santos, J. R., Haimes, Y. Y., Lian, C., 2007. "A Framework for Linking Cybersecurity Metrics to the Modeling of Macroeconomic Interdependencies", Risk Analysis, Vol. 27, No. 5.

Savage, L. J., 1954. *The Foundations of Statistics*, John Wiley & Sons, New York, NY.

Shanteau, J., Weiss, D. J., Thomas, R., Pounds, J., 2001. "Performance-based Assessment of Expertise: How to Decide if Someone is an Expert or Not", *European Journal of Operations Research*, 136, 253-263.

Stevens, S. S., 1946. "On the Theory of Scales of Measurement" *Science*, vol. 103, pp. 677-680.

von Bertalanffy, L., 1968. *General Systems Theory, Foundations, Development, Applications*, University of Alberta, Edmonton, Canada, published by George Braziller, One Park Avenue, New York, New York, 10016.

von Neumann J., Morgenstern O., 1944. *Theory of Games and Economic Behavior*, Princeton University Press, Princeton, New Jersey 08540.

von Winterfeldt D., and Edwards, W., 1986. *Decision Analysis and Behavioral Research*, Cambridge University Press, Cambridge, United Kingdom.

Weisstein, Eric W. "Graph." From MathWorld: A Wolfram Web Resource. [mathworld.wolfram.com/Graph.html](http://mathworld.wolfram.com/Graph.html).

White, B. E., 2006. "Fostering Intra-Organizational Communication of Enterprise Systems Engineering Practices", The MITRE Corporation, National Defense Industrial Association (NDIA), 9th Annual Systems Engineering Conference, October 23-26, 2006, Hyatt Regency Islandia, San Diego California.