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On Modeling and Mitigating Financial Bubbles

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Abstract

Financial "bubbles," show a period of strong asset price appreciation followed by a collapse. As witnessed currently, the collapse of a bubble can lead to a loss of jobs and a severe economic contraction. The two question addressed here are: "Can bubbles be modeled sufficiently well so that meaningful forecasts of their impending development can be made well in advance of their collapse and if so, what is the most appropriate action to mitigate their development and ultimately to prevent them from forming?" These questions are explored using a simple model for the demand and pricing of assets, which is tested by seeing how well it can simulate several prior and current bubbles including the ongoing bubble in the U.S. housing market.

1. Introduction

Although the financial gains and losses made by investors over the life of a bubble tend to cancel out when summed over all investors, significant societal suffering, nevertheless, occurs from the general economic malaise and job loss that follows. The explanation of why the net impact is not zero but very negative comes from Prospect Theory [1], which has shown that the loss in utility for a monetary loss can be two to three times the amount for an equivalent monetary gain. Two central questions are considered here. Question 1 is: "Can bubbles be modeled sufficiently well so that meaningful forecasts of their development can be made well before the bubble is to collapse?" Question 2 is: "If so, what is the most appropriate action, to mitigate and ultimately to prevent their formation?"

The Federal Reserve Bank (Fed) [2] has spoken to the need to better understand the elements, both financial and human, that drive bubbles. The concern is broad. Premier Wen Jiabao of China is on record of expressing the view that asset bubbles need to be prevented [3]. Shiller [4] has proposed a visionary recommendation for "democratizing finance" as a means to mitigate bubbles. Bernanke and Gertler [5] noted that a key uncertainty in structuring the right action to weaken a bubble is whether the aggressive price increases are a result of widespread speculation or the honest appreciation in the asset's value. The approach taken here to understanding bubbles is to model their behavior using a simple, highly-transparent model that, nevertheless, contains the major elements that are believed to lead to the formation and collapse of bubbles.

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2. Models for Financial Bubbles

Models used for financial bubbles can be placed roughly into one of three types:

value-based phenomenological (VBP),
agent-based algorithmic, and
log-periodic critical.

None of the models support the classical concept of a highly efficient market in which the price of an asset always reflects its current intrinsic value. The VBP model has two contributions to the net perceived value of an asset, its intrinsic value, which may or may not vary appreciably over time, plus a "chartists" value computed from price trends prior to the current time. This approach has been used by Beja and Goldman [6], Chiarella [7], Levy *et. al*.[8], Farmer and Joshi [9], Malecky′ and Sergyeyev [10], and Levy *et. al*. [11]. Malecky′ and Sergyeyev [10] studied the price behavior of two individual stocks over 30 to 60 days. Price trends used in the computing the chartist contributions were taken at 10 to 20 days prior to the date of a specific price being modeled. Reasonable agreement was found with actual price behavior but bubbles were not studied. Farmer and Joshi [9] simulated the logarithm of the prices of S&P composite index over a very large range of time starting from before year 1900 and going somewhat past 1980. They found qualitative agreement between the simulated and actual prices. They used a rough, time-dependent computation of intrinsic value based upon dividends. The agent-based algorithmic approach was pioneered by the Santa Fe Institute using genetic algorithms [12, 13]. Hong, *et. al*. [14] modeled bubbles using one set of agents as advisors to another set who represented investors. There is considerable interest in the physics community in modeling the price dynamics of bubbles using log-periodic oscillations exhibiting critical-like behavior. Major contributions to this line of research include the papers by Feigenbaum and Freund [15] and Sornette *et. al.*[16]. Sornette [17] has given an excellent account of this approach but some details remain unsettled at this juncture [18, 19, 20].

3. The Approach Here

The approach here to answering the two questions raised earlier is in three steps :

Step 1: Model (simulate) real bubbles.
Step 2.: Use the insight gained to identify their root cause.
Step 3.: Seek informed answers to the two questions raised.

Human behavior and judgment are largely if not fully responsible for the behavior of real markets. For example Gennotte and Leland [21] reasoned that few investors "gathered information on future economic prospects or asset supply." This represents an example of what Simon [22] termed satisficing whereby we tend to use simple heuristics instead of working through the proper but more complicated and time-consuming analysis. The tendency is to sense, often improperly, if the price is going up, then intrinsic value is going up at the same rate.

Consequently, a highly transparent VBP demand and pricing formalism [23, 24] is used here to model the bubbles because it is able to quantify the key elements of human behavior that influence bubbles. The model, which has the ability to simulate with ease the quasi-lambda form exhibited by most bubbles, assumes that both buyers and sellers add speculative value to the asset's intrinsic value in proportion to the rate of change of price with time measured at a time prior to the transaction. Thus in the euphoria of a boom, speculative value drives up the price of the asset. In the gloom of a bust, it drives the price down.

The bubbles that have been simulated using this approach are the 1928-1929 Dow Jones Industrials (DJI) composite index, the current U.S. housing market, the Nasdaq composite index at the turn of this century, the current S&P 500 composite index, and the 1987 crash of the DJI composite index. The model uses two different populations called "buyers" and "sellers," but each, will buy or sell an asset if the price is right. Buyers have an intrinsic value of $V_{0,B}$ for the asset which is higher than the intrinsic value of sellers $V_{0,S}$. The average of the two intrinsic values represents the stable future price for the asset in the absence of speculation.

4. Dual Buyer-Seller Model

Two hypothetical demand curves are shown in Fig.1. The one for sellers is on the left and the other is for buyers who value the asset higher.[2] The price intercepts at zero demand represent the intrinsic values of $V_{0,S}$ and $V_{0,B}$ for sellers and buyers, respectively, which are \$160 and \$164, respectively, in this example. Cook and Wu [25] showed that the extrapolated demand functions did in fact intercept the zero demand line at the known expected economic values for the simulated purchase of lottery tickets. Because there are just two classes of investors, the model is termed the Dual-Buyer-Seller (DBS) model. No special software is needed to run the model beyond Excel's Solver.

The *perceived* value of an asset is the sum of its intrinsic and speculative value [23, 24]:

$$V_i = V_{0,i} + c\left.\frac{\delta P}{\delta t}\right|_{t^*} \tag{1}$$

where i = S for sellers and B for buyers. The term t^* is the historical time when the price change was evaluated. It is prior to the current time t by an amount Δ, which is evaluated empirically from the fit of the model to the actual prices of in the development and collapse of the bubble. The coefficient c measures the strength of speculation.

Demand is equal to perceived value minus price, the quantity times the negative slope K of the demand curve:

[2] The units shown for demand are arbitrary. Positive demand is the rate at which assets are offered for purchase per unit time and negative demand is the rate at which assets are offered for sale per unit time.

$$D_i = K\left[V_{0,i} + c\left.\frac{\delta P}{\delta t}\right|_{t^*} - P\right] \tag{2}$$

Time, *t*, is an integer equal to the number of trading days measured from an earlier reference point and the prices $P = P(t)$ are taken as the closing prices over the time range of interest. Market equilibrium according to the model is defined by a stable price equal to P_{Eq} defined as the average of the two intrinsic values:

$$P_{Eq} \equiv \left[V_{0,B} + V_{0,s}\right]/2 \tag{3}$$

P_{Eq} *is the "invisible hand" that tries to pull undervalued and overvalued assets back to their intrinsic value*. If price varies with time, then some degree of speculation exists if $c \neq 0$ and the current price is not given by Eq. (3). There can be a remarkable positive departure of price from P_{Eq} on the boom side of a bubble as market euphoria pushes speculative value (and price) much higher than the intrinsic value of the asset. The absolute value of the speculative contribution dominates the intrinsic value in the region about the bubble's peak. This dominance is likely why the simple model is able to simulate the price dynamics of bubbles in spite of the assumption of a constant intrinsic value

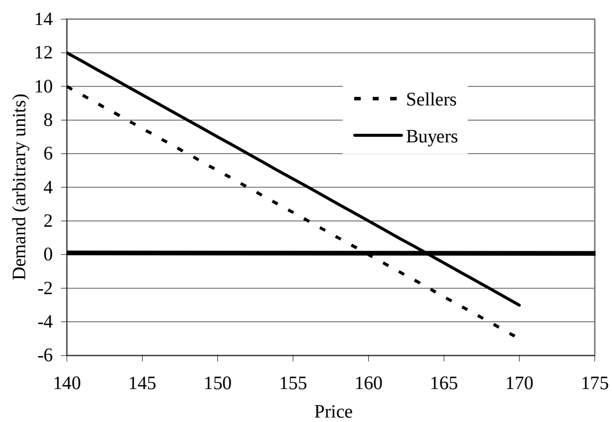

Fig. 1. The demand curves for sellers and buyers. The intercepts with the zero demand line represent the intrinsic values of the stock to the sellers and buyers.

The finite difference equation for price can be written as [24]:

$$\begin{aligned} P(t+\delta t) - P(t) &= a'P(t)\delta t\left[D_S + D_B\right]/N_{SH} \\ &= aP(t)\delta t\left[P_{Eq} + c\left.\frac{\delta P}{\delta t}\right|_{t*} - P(t)\right] \end{aligned} \tag{4}$$

where:

$$\left.\frac{\delta P}{\delta t}\right|_{t^*} = \frac{P(t-\Delta+\delta t)-P(t-\Delta)}{\delta t} \tag{5}$$

For a stable price, it follows from Eq. (4) that $D_B = -D_S$. For stocks δt is usually one trading day ($\delta t = 1$) with the price being the adjusted closing price. The parameter $a'$is a dimensionless coupling constant, N_{SH} is the number of shares outstanding, and $a = 2a'K / N_{SH}$. Subordinate peaks in the simulated prices occur about the dominant peak for the bubble, being especially strong during the first part of the collapse with spacings approximately equal to Δ.

It was immediately discovered in making simulations that two speculative coefficients were required to generate an accurate simulation. Consequently, a coefficient defined as $c1$ was used for the boom side and $c2 = c1/ f$ was used for the bust. Thus, the model quantifies four human behavioral variables, the boom and bust speculation coefficients, the average intrinsic value, and the prior time Δ. When f is greater than unity, $c2$ is smaller than $c1$ and the speculative pressure sensed by traders to sell an asset when prices are falling is not as large as the speculative pressure sensed by them to purchase the same asset when prices were rising. This form of risk taking is not uncommon for individuals when faced with a potential loss [1, 26].

5. Analysis of Actual Bubbles

5.A. 1928-1929 Bubble in the Dow Jones Industrials Composite Index

The comparison of the simulated and actual prices for the Dow Jones Industrials composites index (symbol DJI) is shown in Fig. 2. The first trading day (t-day with 297 t-days per year on average during this period) is January 2, 1925 and the highest peak in price is for September 3 (t-day 1393). The Black Monday collapse was on t-day 1439. The parameters generated by Solver's optimization are also shown on the graph. The intrinsic value of \$168 (the coefficient b is P_{Eq}) indicates that the market was already overvalued by September 1927. The maximum in speculative value given by $c1\times dP/dt$ at t^* up to and including the peak was \$498, three times intrinsic value, and occurred at t-day 1360. The RMS deviation between the simulated and actual prices is \$21.51, which is 12% of the intrinsic value and 5.7% of the peak price. All of the empirical coefficients found were significant at better than 1%. This was true of all of the bubbles analyzed.

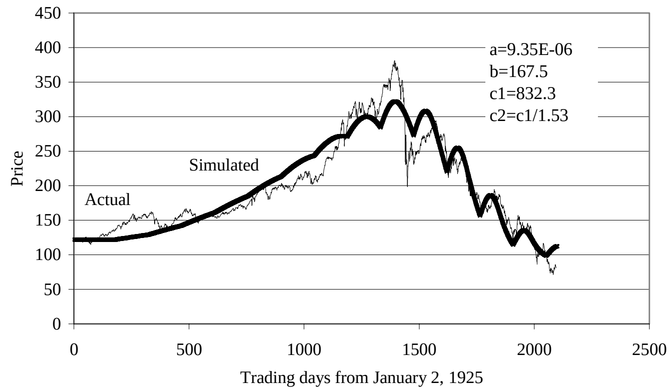

Fig. 2. The actual and simulated prices versus *trading days* (t-days with 297 per year for this period) for the 1928-1929 DJI composite index bubble.

A t-day prior time of 144 was selected for Δ because it gave the smallest RMS price deviation. Although this prior time may appear to be large, it is well-established that the distribution of stock prices over time exhibit "long term memory" [27, 28, 29]. The factor f was found to be 1.53 which indicates that speculative selling on the downside of the bubble was not as strong as the speculative buying on the upside of the bubble. As already discussed, risk-taking behavior during downturns, expressed here as a reluctance to sell quickly at the start of the collapse, has been well-documented in other venues by Shefrin and Statman [26].

5.B. Analysis of the Current U.S. Housing Bubble

The current U.S. housing bubble, Fig. 3, peaked at $264,650 in the 1st quarter of 2006. (The prices shown are for the median priced U.S. home in inflation adjusted dollars from a 1st quarter 2009 price base divided by 100,000.) The reduced actual prices are shown over 57 quarters extending from January 1, 1995 to the 1st quarter of 2009 (5,155 annual days at 365 per year). The simulated prices extend to 110 quarters. The simulation yields a classic lambda shape. The minimum of $134,000 is projected to occur in the 1st quarter of 2012. The inflation adjusted intrinsic value is only 61 % of the peak price. A prior time of $\Delta = 5$ quarters was used in making the simulations. The RMS error between the simulated and actual prices was $4,728 which is 3% of intrinsic value. The factor f was found to be 2.1 indicating that participants in the market were more eager to buy with prices rising than to sell when prices started to fall. In Fig. 3, the simulated price arrives at the inflation adjusted intrinsic value of P_{Eq} = $162,000 during the 2nd quarter of 2018.[3] The extrapolation shown was made with the bust-side speculation coefficient equal to c2. If, however, home buyers did not learn from the crash in home prices and begin to speculate as prices start to rise with the larger coefficient equal to c1 then another bubble is projected to form.

[3]Minimal confidence is placed in the extension of the simulated results for housing prices well-beyond the actual prices as the extrapolation error of the non-linear model can be large. It also ignores the possibility of effective Federal stimulation of the housing market. Nevertheless, it is somewhat sobering of what could happen if judged with this caveat.

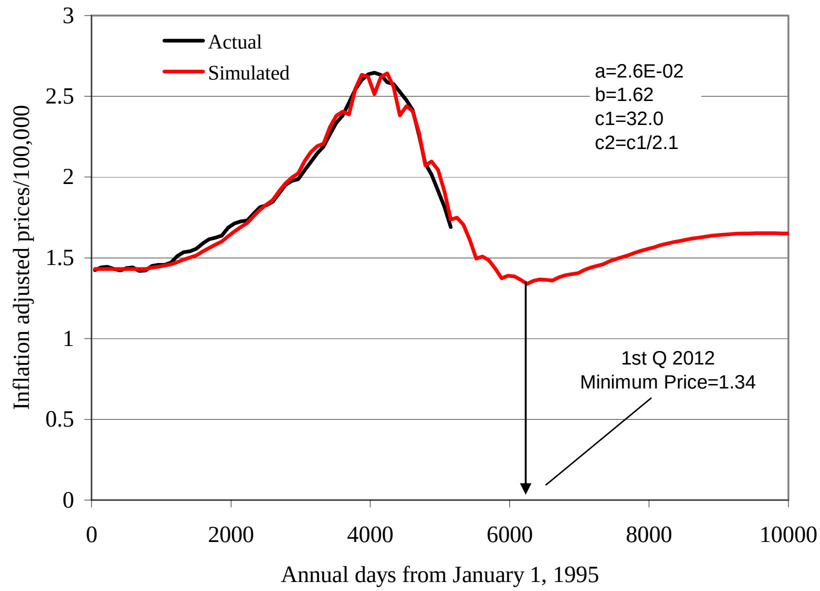

Fig. 3. The actual and simulated inflation adjusted median U.S. home prices divided by 100,000.

During the 3rd quarter of 2005, Neil Henderson [30] of the *Washington Post* reported that Ben S. Bernanke[4] testified to Congress that the marked increases seen in housing prices "largely reflect strong economic fundamentals." Economists who felt otherwise were correct as housing prices are currently well into the downside of the bubble as shown in Fig. 3. Thus it is relevant to ask whether or not there was sufficient information in housing price dynamics prior to the 3rd quarter of 2005 to predict convincingly the likelihood of the crash.

Fig. 4 was constructed using housing prices only up to the 4th quarter of 2003. It shows the likelihood of a bubble being forecast well in advance of Bernanke's comments that the rise in housing prices was not due to the formation of a bubble. Zhou and Sornette [31] also predicted a bubble for the U.S. market using prices truncated at the 1st quarter of 2005 based upon their position that a faster-than-exponential rate for price is a predictive criteria for a bubble. The relatively quick formation a second bubble in Fig. 4 is an artifact caused by setting c2 on the collapse side of the simulation equal to c1 instead setting it equal to the reduced size found in Fig. 3 of c2=c1/2.1. (This choice was made to be more representative of what an analyst might assume when simulating a bubble with prices available only on the boom side.)

5.C. Other Bubbles

In regard to predicting other bubbles well before the collapse, a bubble was forecast for the 1929 DJI bubble using prices truncated at January 9, 1929, which was 239 t-days before October 28, 1929. A bubble was also predicted for the current S&P 500 using a truncated price range from July 12, 2006 (t-day 1139) to July 24, 2007 (t-day 1398), the latter being 54 t-days before the peak on October 9, 2007.

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[4] Just prior to being appointed Chairman of the Federal Reserve.

By contrast, the model did not forecast an imminent bubble before the October 19, 1987 "Black Monday" crash of the DJI composite index even when the upper limit of the price range was taken close to October 19. However, using prices only up to April 9, 1987 (133 t-days before the crash) a superheated intrinsic value of $6,807 was found, which was 2.9 times the price of $2,339 on April 9. Similarly the model did not forecast a bubble for the 2000 Nasdaq crash using prices truncated just before the very sharp run-up in price that formed the top of the bubble. But again, the intrinsic value found was 2.6 times the current price. Thus, when an intrinsic value is found that is much larger than the current price, it appears to signal that prices are rising at an unsustainable rate and that a major correction is likely in the not too distant future. This conjecture, of course, is based upon limited findings and needs to be supported by a wider study of historical and future bubbles. If validated, then this model signals that a major correction may be imminent not only when the simulation clearly forecasts a bubble in the future as in Fig. 4 but also when it yields an intrinsic value greater than 2.5 times the current price. When used in conjunction with the other predictive models described earlier, their collective insights should provide a robust early warning system for detecting bubbles before they reach a critical stage.

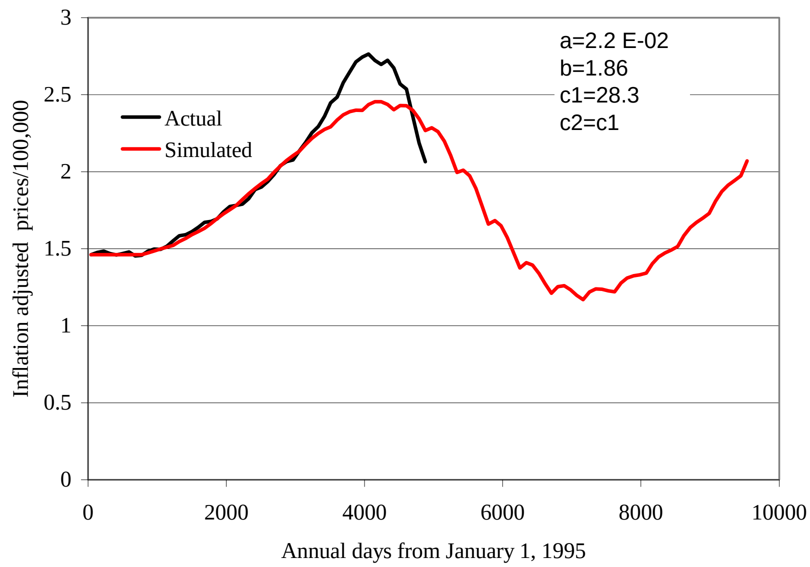

Fig. 4. This simulation of the housing bubble was computed using actual price information only up to the 4th quarter 2003 (a-day 3239).

6. Mitigating and Preventing Bubbles

The bubbles studied here strongly support the view that asset prices often do not reflect their intrinsic value because many traders use price dynamics to arrive at their perceived value of the asset instead of working through the financial task of assessing intrinsic value [21]. Value investors likely buy at the start of a bubble, but when the boom is well underway, price-dynamics speculators enter and bid up the assets until their prices are well overvalued. The simulation results are not inconsistent with this view as the speculative component of perceived value was relatively small at the outset due to price being relatively flat but it greatly increased as the boom progressed. Hong, et. al. [14] have presented strong evidence that specialists issuing “optimistic forecasts to naïve investors” encouraged price-dynamics speculation during much of the year 2000 Nasdaq price appreciation.

Without strong historical support, warnings about the existence a bubble and its imminent collapse may not work. For example, PINR [32] reported on an interview held on January 30, 2007 between the *Financial Times* and Cheng Siwei, Vice Chairman of the Standing Committee of the National People’s Congress, who said among other things that “There is a bubble going on.” The Shanghai Stock Exchange dropped 11% after the comment but it had already started a recovery by February 6 and soon “surpassed a previous record high.” This warning simply provided a dip that was taken as a buying opportunity by others. There is little evidence that Alan Greenspan’s “irrational exuberance” remark on December 6, 1996 [33] gave the market pause. The warning on March 8, 1929 (t-day 1246 in Fig. 2) by Paul M. Warburg, famed banker and member of the first Fed, shows no obvious impact on the price dynamics.

The answer given here to Question 1 is to use the repeated modeling of a developing bubble to provide a robust early warning system. Several predictive models are available and should be used in concert to provide the robustness and broad insight needed. With this information there are various paths that a mitigation process could take. For example, the Fed could use the warnings to revise interest rates and Congress could weigh-in through tax policy thereby following the standard route of using monetary and fiscal policies to cool the economy. Lansing [2] has reported that the Fed is reconsidering how monetary policy could be used to mitigate bubbles and that the price of the S&P 500 composite index has influenced the Fed’s discount rate in the past. Based on additional remarks by Lansing, routine use of monetary policy by the Fed to mitigate bubbles appears unlikely until there is confidence that it can identify a developing bubble.

Also during the euphoria of a bubble, traders may not react to such policies. For example, the Fed raised the discount rate from 5 to 6% on February 14, 1929 [34] (t-day 1229, Fig. 2), but the evidence is not that strong that it had much of an influence on the bubble as the DJI composite index increased by an additional $72 (24%) to reach its peak on September 3, 1929 (t-day 1393 in Fig.2). Some blamed the crash on the Fed’s hike in rates even though, as already discussed, all of the parameters were in place by January 9, 1929 to forecast the bubble’s peak and downturn. Thus a legitimate concern is that the initiator of a mitigation policy could be identified as the perpetrator causing the crash rather than the enlightened agency that generated the needed controlled burn which avoided a much more cataclysmic event. Finding the right rate is critical; too little and the bubble goes on; too much and the market could collapse into a recession. So there are issues, but monetary and fiscal policies have a larger issue, they do not attack the problem’s root cause of *poorly informed traders*.

A private, not-for-profit company could be formed to address the root cause using a two step process of mitigation and prevention. Mitigation policy would be based upon highly credible warnings backed up by extensive studies of prior bubbles. Most future bubbles could be *prevented* if price-dynamics speculators come to understand that it is in their interest to become value investors. This is the answer suggested here to Question 2 and it follows Shiller's [4] call to "democratize finance."[5]

Based upon the arguments given above, the following actions are suggested as an approach to mitigating bubbles and to reducing the number formed over time. It is assumed that actions are carried out by a not-for-profit company named NFP:

1. NFP initiates a high-quality educational program for wide distribution that describes the origins of asset bubbles and their negative consequences. Examples of bubbles through history should be used to illustrate what the causes were and how investors and the economy were affected by the crash.
2. NFP would examine price appreciations of major assets including stocks and housing and would issue warnings as appropriate when the development of a bubble is detected.
3. NFP would improve the speed, accuracy, and generality of the models and tools used to predict bubble formation, intrinsic value, and speculative value.

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References

[1] D. Kahneman, A. Tversky, "Prospect Theory: An Analysis of Decisions Under Risk," *Econometrica* Vol. 47, 1979, pp. 263-291; A. Tversky, D. Kahneman, "Loss Aversion In Riskless Choice A Reference-Dependent Model," *Quarterly Journal of Economics*, Vol. 106, 1991, pp. 1039-1061.

[2] K.J. Lansing, "Should the Fed React to the Stock Market," *FRBSF Economic Letter*, Vol. 34, 2003, pp. 1-3.

[3] Associated Press, http://www.cnn.com/2007/BUSINESS/11/19/china.stock.market.ap/index.htmlS

[4] R.J. Shiller, *The Subprime Solution*, Princeton University Press, Princeton, NJ, 2008, p. 115.

[5] B. Bernanke, M. Gertler, "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review*, Vol. 91, 2001, pp. 253-257.

[6] A. Beja, M.B. Goldman, "On the Dynamic Behavior of Prices In Disequilibrium," *The Journal of Finance*, Vol. 35, 1980, pp. 235-248.

[7] C. Chiarella, "The Dynamics of Speculative Behavior," *Annals of Operations Research*, Vol. 37, 1992, pp. 101-123.

[8] M. Levy, H. Levy, S. Solomon, "A Microscopic Model of the Stock Market:Cycles, Booms, qnd Crashes," *Economics Letters*, Vol. 45, 1994, pp. 103-111.

[9] J.D. Farmer, S. Joshi, "The Price Dynamics of Common Trading Strategies," *Journal of Economic Behavior & Organization*, Vol. 49, 2002, pp. 149-171.

[5] R.J. Shiller's position on financial democratization is that "sound financial principles" and sound financial information are mostly absent in today's markets and need to become available all investors current and potential.

[10] J. Melecky, A. Sergyeyev, “A Simple Finite-Difference Stock Market Model Involving Intrinsic Value,” *Chaos, Solitons And Fractals*,” Vol. 38, 2008, pp. 769-777.

[11] M. Levy, N. Persky, S. Solomon, “The Complex Dynamics of a Simple Stock Market Model,” Racah Inst. Of Physics, Hebrew University, Jerusalem, 2002

[12] W.B. Arthur, J.H. Holland, B. LeBaron, R. Palmer, P. Taylor, “Asset Pricing under Endogenous Expectations, in an Artificial Stock Market,” in *The Economy as an Evolving Complex System* II, W. Arthur, D. Lane, and S. Durlauf editors, Addison-Wesley, Redwood City, 1997, pp. 287-300.

[13] B. LeBaron, “Building the Santa Fe artificial stock market,” Brandeis University, 2002.

[14] H. Hong, J. Scheinkman, W. Xiong, “Advisors and Asset Prices: A Model of the Origins Of Bubbles,” *Journal of Financial Economics*, Vol. 89, 2008, pp. 268-287.

[15] J.A. Feigenbaum, P.G.O. Freund, “Discrete Scale Invariance in Stock Markets Before Crashes,” *International Journal Modern Physics*, Vol. B 10, 1996, pp. 3737-3740.

[16]].D. Sornette, A. Johansen, J-P. Bouchard, “Stock Market Crashes, Precursors, and Replicas,” *Journal Physique*, Vol. 6, 1996, pp. 167-175.

[17] D. Sornette, *Why Stock Markets Crash*, Princeton University Press, Princeton, 2003.

[18] J.A Feigenbaum, “A Statistical Analysis of Log-Periodic Precursors to Financial Crashes,” *Quantitative Finance*, Vol. 1, 2001, pp. 346-360.

[19] D. Sornette, A Johansen, “Significance Of Log-Periodic Precursors To Financial Crashes,” *Quantitative Finance*, Vol.1, 2001, pp. 452-471

[20] J.A Feigenbaum, “More on a Statistical Analysis of Log-Periodic Precursors To Financial Crashes,” *Quantitative Finance*, Vol. 1, 2001, pp. 527-532

[21] G. Gennotte, H. Leland, “Market Liquidity, Hedging, and Crashes,” *The American Economic Review*, Vol. 80, 1990, pp. 999-1021.

[22] H.A. Simon, “Theories of Decision-Making in Economics and Behavioral Science,” *American Economic Review*, Vol. 49, 1959, pp. 253–283.

[23] H.E. Cook, *Product Management, Value, Quality, Cost, Price, Profits, And Organization*, Kluwer Academic, Amsterdam, 1997, pp. 48-54.

[24] H.E. Cook, L.A. Wissmann, *Value Driven Product Planning and Systems Engineering*, Springer-Verlag London, 2007, pp. 85-90.

[25] H.E. Cook, A. Wu, “On The Valuation of Goods And Selection of The Best Design Alternative,” *Research in Engineering Design*, Vol. 13, 2001, pp. 42-54.

[26] H. Shefrin, M. Statman, “The Disposition to Sell Winners too Early and Ride Losers too Long: Theory and Evidence,” *The Journal of Finance*,Vol. 3, 1985, pp. 777-790.

[27] K. Ide, D. Sornette, “Oscillatory Finite-Time Singularities in Finance, Population and Rupture,” *Physica A*, Vol 307, 2002, pp. 63-106.

[28] Z. Ding, C.W.J. Granger, R.F. Engle, "A Long Memory Property of Stock Market Returns and a New Model," *Journal of Empirical Finance*, Vol. 1, 1993, pp. 83-106.

[29] A. Johansen, D. Sornette, "The Nasdaq Crash Of April 2000: Yet Another Example od Log-Periodicity in a Speculative Bubble Ending in a Crash," *The European Physical Journal*, Vol. B 17, 2000, pp. 319-328.

[30] N. Henderson, Bernanke: there's no housing bubble to go bust, Washington Post, October 27, 2005.

[31] W-X Zhou, D. Sornette, "*Is There a Real Estate Bubble in the U.S.?*," *Physica A*, 2006, pp. 297-308.

[32] Power and Interest News Report (PINR), Economic Brief: China's Stock Market Bubble, February 19, 2007.

[33] C.P. Kindleberger, R. Aliber, *Manias, Panics, and Crashes*, 5th ed., Wiley, New York, 2005, p. 18.

[34] R.J. Shiller, *Irrational Exuberance*, 2nd ed., Doubleday, New York, 2005, pp. 224-225.